ADAPTIVE HARMONIC COMPONENTS DETECTION AND FORECASTING IN WAVE NON-PERIODIC TIME SERIES USING NEURAL NETWORKS

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Abstract. The identification and forecasting problem of non-periodic time series with wave structure and the problem of latent periodic component detection in stochastic time series are considered. The adaptive forecasting method is proposed using the special autoregression representation of wave time series and both frequencies and amplitudes of partial harmonics identification. An artificial neural network comprising band-pass digital filters and a generalization unit is designed that allows the real-time extraction of an arbitrary number of harmonic components from the analyzed signal. Learning algorithms for the proposed architecture are developed. Model simulation results are presented and the results of real problem solution of stock prices forecasting and trading decision support are also considered.

Key words: identification, financial engineering, forecasting, neural networks, signal processing, time-series.

1 Introduction

The necessity of complex structure time series forecasting takes place in many problems of automatic control, signal processing, econometrics and so on [1]. The efficiency of forecasting essentially depends from the adequacy of time series model. The popular forecasting methods usually use the simple models like "trend + noise" or ARMA models in combination with recurrent parameters identification algorithms [2,3]. In practice, however, the real time series have a more complex structure such as non-periodic oscillating time series. Even the simple superposition of a number of harmonics with aliquant frequencies leads to the time series structure mentioned above. Such a time series, so calls "wave" time series, are widely used as mathematical models of disturbances in control systems, seasonal processes in economics [4], stock prices in financial engineering and in other different applications [5].

The identification problem of such time series is considerably simple in the case, when the wave component is the superposition of a number of harmonics with known frequencies and phases, at that the unknown amplitudes identification may be performed by simple algorithms like recurrent least square method. The problem has become more complicated when both frequencies and phases are arbitrary unknown and changing in time. In general case such time series are non-periodic and unknown frequencies extraction by the Discrete Fourier Transformation (DFF) methods [6] does not lead to the good result.

Moreover, in many signal processing related applications a problem of periodic components detection from a noise-disturbed signal arises. Such a problem is usually reduced to estimation of harmonic component parameters against background of noise and solved using traditional DFF methods. However, when non-periodic oscillating time series signal processing is required, some additional problems arise and alternative to the DFF methods based on adaptive digital filters [3,7] are more frequently used.

In this paper an adaptive neural networks based approach for wave time series forecasting is considered based on a special assignment of wave component auto-regression model as a superposition of harmonics with tuning frequencies. In such a case, the suitable identification algorithm ensures non-stationary frequencies tracking. The proposed method based on the structural modeling approach [8,9] is seemed to be very useful for seasonal and oscillating economic time series. The periodic components detection method based on neural network techniques [10] is also proposed.

2 Wave time-series identification

Consider the trend-seasonal time series model

$$Y_k = \sum_{i=0}^{n} D_i k^i + \sum_{j=0}^{m-1} \left[A_j \cos \omega_j k + B_j \sin \omega_j k \right] + \xi_k \qquad (1)$$

where Y_k - the time series value at instant k, n - the degree of polynomial component, m - number of harmonics with frequencies $0 < \omega_j = 2\pi f_j T_0 < \pi$, T_0 - sampling period, ξ_k - random zero mean measurement noise. The time series forecasting is based the identification of mathematical model parameters D_i, A_j, B_j, ω_j. Because the identification of trend and wave components performs by different methods, at first it is necessary to divide the components of time series. At that the trend component may be extract by the following two methods.

A) Discrete smoothing method.

In such a case trend is extracted by means of law-pass filter realized by exponential smoothing algorithm. Let $n = 0$, i.e. trend is a constant bias D_0. Then using the smoothing procedure

$$\bar{Y}_k^s = \alpha_s \bar{Y}_{k-1}^s + (1-\alpha_s) Y_k, \quad 0 < \alpha_s < 1, \qquad (2)$$

the initial time series (1) can be divided to the slow component - trend estimation $\bar{Y}_k^s$ and the fast one $\tilde{Y}_k = Y_k - \bar{Y}_k^s$, which is the linear transformation of the wave component distorted by random noise $\tilde{\xi}_k$,

$$\tilde{\xi}_k = \xi_k - \bar{\xi}_k^s, \quad \bar{\xi}_k^s = \alpha_s \bar{\xi}_{k-1}^s + (1-\alpha_s)\xi_k. \qquad (3)$$

It is evident that the smoothing transformation does not change the wave component spectrum. In general case $n > 0$ it is possible to apply the high order exponential smoothing.

B) Discrete differentiation method

In such a case the trend extraction is performed by discrete differentiation of time series (1), which is previously smoothed by discrete wide-band filter in the purpose of noise reduction:

$$\begin{aligned} \tilde{Y}_k^d &= \alpha_d \tilde{Y}_{k-1}^d + (1-\alpha_d) Y_k, \quad 0 < \alpha_d < 1, \\ V_k &= \tilde{Y}_k^d - \tilde{Y}_{k-1}^d = (1-\alpha_d)\left(Y_k - \tilde{Y}_{k-1}^d\right) \end{aligned} \qquad (4)$$

As a result the first difference sequence V_k has the structure of wave component distorted by the equivalent noise

$$\begin{aligned} v_k &= \frac{1-\alpha_d}{\alpha_d}\left(\xi_k - \tilde{\xi}_k^d\right), \\ \tilde{\xi}_k^d &= \alpha_d \tilde{\xi}_{k-1}^d + (1-\alpha_d)\xi_k. \end{aligned} \qquad (5)$$

At that both methods lead to the identification problem of wave time series

$$y_k = \sum_{j=0}^{m-1} \left[a_j \cos \omega_j k + b_j \sin \omega_j k \right] + \zeta_k, \qquad (6)$$

where $y_k = \tilde{Y}_k$, $\zeta_k = \tilde{\xi}_k$ in case (a) and $y_k = V_k$, $\zeta_k = v_k$ in case (B).

Using z - transformation, the model (6) may be presented in the form:

$$\prod_{j=0}^{m-1} \left[1 - 2\cos\omega_j z^{-1} + z^{-2} \right] y_k = \zeta_k. \qquad (7)$$

Realizing the inverse transformation the equation (6) may be representing in time domain in the linear auto-regression form:

$$y_k = \sum_{j=0}^{m-1} \beta_j \left(y_{k+j-m} + y_{k-j-m}\right) - y_{k-2m} + \zeta_k = \\ = \beta^{\mathrm{T}} y(k,m) - y_{k-2m} + \zeta_k, \quad (8)$$

where $y(k,m) = (2y_{k-m},\ \ y_{k-m+1} + y_{k-m-1}, \ldots, y_{k-1} + y_{k-2m+1})^{\mathrm{T}}$ is the time series "prehistory" vector, $\beta^{\mathrm{T}} = (\beta_0, \beta_1, ..., \beta_{m-1})$ - model parameters.

Using the quadratic identification criterion

$$J = \sum_{k=2m}^{N-1} \left[y_k + y_{k-2m} - \beta^{\mathrm{T}} y(k,m)\right]^2 \quad (9)$$

one can obtain the recurrent algorithm of the model identification:

$$\hat{\beta}_k = \hat{\beta}_{k-1} + [y_k + y_{k-2m} - \\ \beta^{\mathrm{T}}_{\mathrm{k\text{-}1}} y(k,m)] y(k,m) r_k^{-1}, \\ r_k = \gamma_k r_{k-1} + \left\| y(k,m) \right\|^2, \quad 0 < \gamma_k < 1, \quad (10)$$

where tuning parameter γ may be used for trade-off adjusting between tracking and flittering properties of the algorithm (10). Using the non-parametric criterion of time series property changing, the following tuning algorithm may be used:

$$\gamma_k = \gamma_{k-1} + \Delta\gamma, \quad \left| \sum_{i=k-q}^{k} \mathrm{sign}\left(y_i - \hat{y}_i\right) \right| \le \delta, \\ \gamma_k = \gamma_{k-1} - \Delta\gamma, \quad \left| \sum_{i=k-q}^{k} \mathrm{sign}\left(y_i - \hat{y}_i\right) \right| > \delta, \quad (11)$$

where q is a memory depth, δ и $\Delta\gamma$ - parameters,

$$\hat{y}_i = \hat{\beta}^{\mathrm{T}}_{\mathrm{i\text{-}1}} y(i,m) + y_{i-2m} \quad (12)$$

is an optimal one step prediction of time series obtained by current estimates.

Frequencies ω_j are connected with parameters β_j by the equation

$$\beta_0 + \sum_{j=1}^{m-1} \beta_j \cos j\omega = \cos m\omega. \quad (13)$$

Taking into account that

$$\cos m\omega = \cos^m \omega - C_m^2 \cos^{m-2} \omega \sin^2 \omega + \\ + C_m^4 \cos^{m-4} \omega \sin^4 \omega + \ldots, \quad (14)$$

frequencies ω_j may be determined as a roots of power polynomial from the argument $\cos\omega$.

Similarly at any instant k the estimations of wave component harmonics amplitudes $\Theta = (a_0, a_1, \ldots, a_{m-1}, b_0, b_1, \ldots, b_{m-1})^{\mathrm{T}}$ may be obtained by the quadratic identification criterion minimization

$$J_\Theta = \left\| Y(k,m) - \Phi(k,m)\Theta \right\|^2, \quad (15)$$

where the extended vector $Y(k,m)$ and matrix $\Phi(k,m)$ are defined as

$$Y(k,m) = \begin{bmatrix} y_{2m} \\ y_{2m+1} \\ \vdots \\ y_k \end{bmatrix},$$

$$\Phi(k,m) = \begin{bmatrix} \cos 2m\hat{\omega}_0 & \ldots & \cos k\hat{\omega}_0 \\ \vdots & \ldots & \vdots \\ \cos 2m\hat{\omega}_{m-1} & \ldots & \cos k\hat{\omega}_{m-1} \\ \sin 2m\hat{\omega}_0 & \ldots & \sin k\hat{\omega}_0 \\ \vdots & \ldots & \vdots \\ \sin 2m\hat{\omega}_{m-1} & \ldots & \sin k\hat{\omega}_{m-1} \end{bmatrix}$$

and $\hat{\omega}_j$, $j = \overline{0, m-1}$ are the frequencies estimations. As a result

$$\hat{\Theta} = \left(\Phi\Phi^{\mathrm{T}}\right)^{-1} \Phi Y. \quad (16)$$

3 Optimal time-series forecasting

Optimal forecast of wave component for p steps $\hat{y}_{k+p}$ may be obtained using the one-step forecast (12) in the form:

$$\hat{y}_{k+p} = \hat{\beta}_k^{\mathrm{T}} \hat{y}(k+p,m) - y_{k+p-2m} \quad p \ge 1, \quad (17)$$

where elements y_i, $k \le i \le k+p-1$ in vector $\hat{y}(k+p,m)$ are replaced by their forecasting values calculated in accordance with (17).

The obtained expressions may be used for initial time series (1) short-term forecasting in accordance with accepted method of components division and structural wave analysis. For method (A) the forecasting formula is

$$\hat{Y}_{k+p} = \overline{Y}_{k+p}^{s} + \tilde{Y}_{k+p}, \quad (18)$$

where $\overline{Y}_{k+p}^{s}$ is a trend component and $\tilde{Y}_{k+p}$ is a wave component forecast.

For the trend component forecasting it is expediently to use the exponential smoothing method. Then

$$\overline{Y}_{k+p}^{s} = \sum_{i=0}^{n} \overline{D}_{ik} p^{i} , \tag{19}$$

where coefficients $\overline{D}_{ik}\left(S_k^0,\ldots,S_k^n\right)$ are expressed from the exponential averages S_k^l, $l = \overline{0,n}$:

$$S_k^l = \alpha_p S_{k-1}^l + \left(1-\alpha_p\right)S_k^{l-1},\ S_k^{-1} = \overline{Y}_k. \tag{20}$$

The wave component forecast is carried out using the proposed technique.

For method (B) taking in account an evident relation:

$$Y_k - \widetilde{Y}_k = \alpha_d \left(1-\alpha_d\right)^{-1} V_k \tag{21}$$

the forecast formula may be represent as:

$$\widehat{Y}_{k+p} = \widetilde{Y}_k^d + \sum_{i=1}^{p} \widehat{V}_{k+i} + \frac{\alpha_d}{1-\alpha_d}\widehat{V}_{k+p}, \tag{22}$$

where first difference forecast is carried out using the proposed frequency estimation method.

4 Harmonic components extraction

In practice for harmonic component extraction from the stochastic time series, non-recursive filters are widely used [8,11,12]. However, as it is noted in [13], the approach based on non-recursive filters is subject to some restrictions especially in the case when the harmonic component frequencies are considerably smaller than the sampling frequency. The point is that if the harmonic components are mixed with a high frequency noise, the noise component during digital processing will be amplified suppressing the original signal. The harmonic components extraction problem may be solved using recursive filters of order 2 tuned on different frequencies [14]. The structure of such a filter is shown on fig. 1 [8], where $\alpha_0, \alpha_1, \alpha_2, \beta_1, \beta_2$ are the tuned filter parameters, $\hat{y}(k)$ - filtered sequence. The recursive adaptive filter structure coincides with the structure of artificial recursive neural network designed to process stochastic signals [11].

In [10] the problem of extraction of m sinusoids with known frequencies $\omega_1, \omega_2, \ldots \omega_m$ from stochastic time series $y(k)$ using a set of band-pass filters is considered. The corresponding structure of the set of filters is shown on fig. 2.

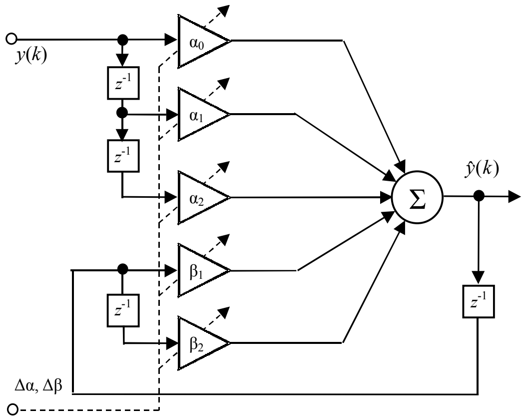


Fig. 1.

Every filter has the transfer function given by

$$F_j(z) = \frac{1-z^{-2}}{1-2\cos\omega_j z^{-1} + z^{-2}} , \tag{23}$$

and the transfer function of the set of filters can be presented as

$$F(z) = \sum_{j=1}^{m} F_j(z) = \sum_{j=1}^{m} \frac{1-z^{-2}}{1-2\cos\omega_j z^{-1} + z^{-2}} \tag{24}$$

Then the filter outputs $\hat{y}_1(k), \hat{y}_2(k), \ldots, \hat{y}_m(k)$ are combined on the base of the stochastic approximation procedure with the gain $1/k$, hereby the output signal is assumed to include only “pure” harmonics.

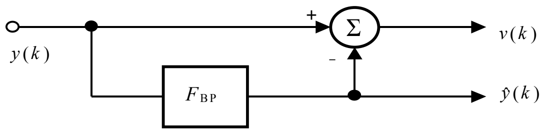


Fig. 2

For unknown frequencies, the so-called “notch” filter [9] was proposed (fig. 3). Here F_{BP} is a band-pass filter with unity gain and zero phase shifts at the resonance frequency. In this case, a particular sinusoid $\hat{y}(k)$ can be extracted by subtraction of the band-pass filter output from its input. Hereby the filter gains for low and high frequencies are the same, so the high frequency noise is not amplified even for very low frequency extracted sinusoids.

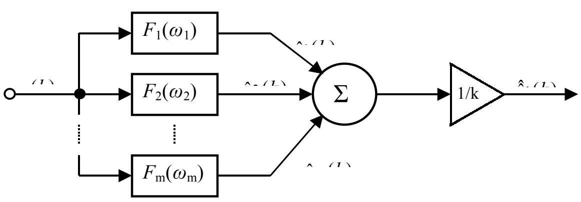


Fig. 3

5 Band-pass biquadrate filter

A band-pass filter can be implemented using a biquadrate element ("biquad") [8,15] shown on fig. 4. Here $\hat{y}(k)$ is the extracted harmonic signal, $v(k)=y(k)-\hat{y}(k)$ is the error signal on the notch filter output, β_1, β_2 are the tuned filter parameters, $s(k)$ is the signal proportional to derivative of the error $v(k)$ with respect to parameter β_2. The transfer function of the band-pass filter

$$F_{BP}(z)= \\ =-\frac{\beta_2}{2}\cdot\frac{1-z^{-2}}{1-\left(2-\beta_2-\beta_1^2\right)z^{-1}+\left(1-\beta_2\right)z^{-2}} \tag{25}$$

is nonlinear with respect to parameter β_1. It allows the resonance frequency tuning with the gain being kept constant for all other frequencies.

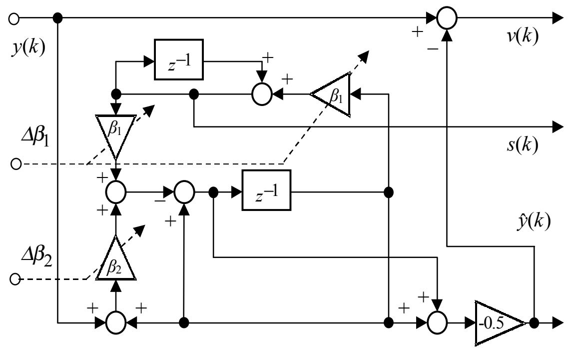


Fig. 4

The resonance frequency of the transfer function (5) is defined as

$$\omega^* = 2\arcsin\left(\frac{\beta_1}{\sqrt{1-\beta_2/2}}\right), \tag{26}$$

and if β_1 and β_2 are small, an approximate estimate is valid

$$\omega^* \approx \frac{\beta_1}{\sqrt{1-\beta_2/2}} \approx \beta_1\left(1+\frac{\beta_2}{4}\right), \tag{27}$$

that is a linear function of β_1 when β_2 is constant. During the extraction of a particular sinusoid from the original signal, β_2 can be kept constant, and tuning is performed only by parameter β_1 variation. After parameter β_1 has been tuned, one can try to tune β_2 to obtain a better selectivity.

6 Filter tuning algorithms

For filter tuning a standard gradient procedure can be used

$$\beta_1(k)=\beta_1(k-1)+\eta v(k)s(k) \tag{28}$$

(here η is a step size parameter). In the general case this procedure coincides with the delta-rule for artificial neural network learning. It should be noted that biquad may be used as an elementary artificial neuron, because the derivative signal is produced by the biquad itself (Fig. 4).

For the case when m sinusoids must be extracted in [9] it was proposed to use a cascade structure containing $m(m+1)/2+m$ tuned biquads. This idea has become the base for the neural network filter shown on fig. 5.

Band-pass adaptive recursive filters used as elementary neurons are tuned with the delta-rule in the following form [9,12]

$$\beta(k)=\beta(k-1)+\eta\frac{v(k)s(k)}{\left\|s(k)\right\|^2}, \tag{29}$$

$$\beta(k)=\left(\beta_1^1(k), \beta_1^2(k), ..., \beta_1^m(k)\right)^T$$
$$s(k)=\left(\partial v(k)/\partial\beta_1^1, \partial v(k)/\partial\beta_1^2, .., \partial v(k)/\partial\beta_1^m\right)^T$$

where j in β_1^j is the corresponding sinusoid number. In the presence of an intensive noise the learning is desired to have further smoothing properties. For this purpose the following nonlinear modification of exponentially weight Goodwin-Ramadge-Caines algorithm [14] was proposed

$$\begin{aligned}&\beta(k)=\beta(k-1)+\eta r^{-1}(k)v(k)s(k),\\ &r(k)=\alpha r(k-1)+\left\|s(k)\right\|^2,\ 0\le\alpha\le 1,\end{aligned} \tag{30}$$

where parameter α defines a trade-off between tracing and smoothing properties of the algorithm.

This structure implements parallel computing and harmonic frequencies can be readily calculated on the base of parameters $\beta_1^j(k)$ using expression (6).

When the learning of elementary filters $F_1, F_2, ..., F_m$ is finished, the network filter outputs $\hat{y}_1(k), \hat{y}_2(k), ..., \hat{y}_m(k)$ produce sinusoids of various frequencies contained in the original signal $y(k)$.

Magnitudes of the harmonics contained in $y(k)$ can be estimated using the generalized prediction

$$\hat{\hat{y}}(k) = \sum_{j=1}^{m} c_j \hat{y}_j(k) = c^T \hat{y}(k), \tag{31}$$

built according to the adaptive multi-model approach [17]. The prediction satisfies the unbiasedness condition

$$\sum_{j=1}^{m} c_j = c^T E = 1 \tag{32}$$

Introducing the generalized prediction error

$$\begin{aligned} w(k) &= y(k) - \hat{\hat{y}}(k) = y(k) - c^T \hat{y}(k) = \\ &= c^T E y(k) - c^T \hat{y}(k) = \\ &= c^T \left(E y(k) - \hat{y}(k)\right) = c^T W(k), \end{aligned} \tag{33}$$

the unknown weighting parameter vector c can be found as a saddle point of the Lagrange function

$$\begin{aligned} L(c,\lambda) &= \sum_{k=3}^{N} c^T W(k) W^T(k) c + \lambda\left(c^T E - 1\right) + \\ & c^T R(N) c + \lambda\left(c^T E - 1\right), \end{aligned} \tag{34}$$

where λ is the indeterminate Lagrange multiplier. The Kuhn-Tucker system solution gives a required estimate

$$c(N) = R^{-1}(N) E \left(E^T R^{-1}(N) E\right)^{-1}, \tag{35}$$

that can be rewritten in the recursive form [15] and is calculated in the output generalization unit G that appears to be m-input adaline [8]. Here $c = (c_1, c_2, ..., c_m)^T$, $\hat{y}(k) = (\hat{y}_1(k), \hat{y}_2(k), ..., \hat{y}_m(k))^T$, $E = (1,1,...,1)^T$ are $(m \times 1)$ vectors

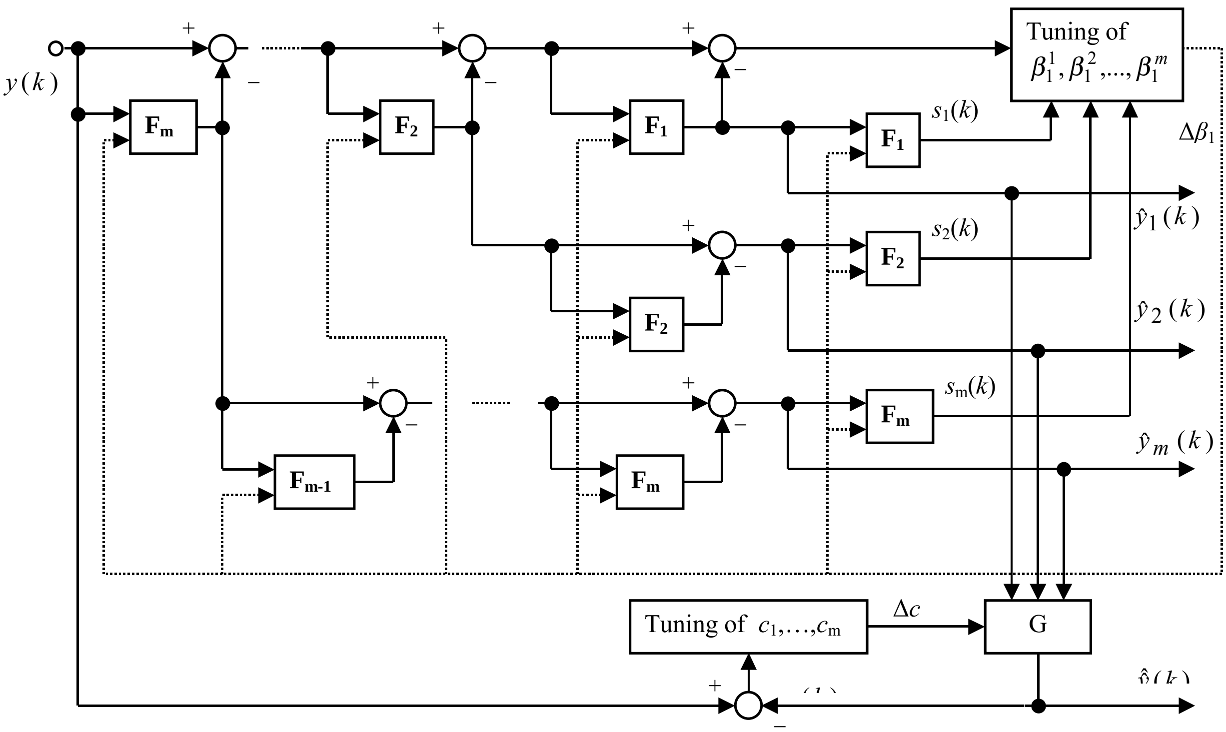
$$\begin{aligned} & P(k) = P(k-1) - \\ & - \frac{P(k-1)\hat{y}(k)\hat{y}^T(k)P(k-1)}{1 + \hat{y}^T(k)P(k-1)\hat{y}(k)}, \\ & c^*(k) = c^*(k-1) + \\ & + P(k)\left(y(k) - \hat{y}^T(k)c^*(k-1)\right)\hat{y}(k), \\ & c(k) = c^*(k) - \\ & - P(k)\left(E^T P(k) E\right)^{-1}\left(E^T c^*(k) - 1\right)E, \end{aligned} \tag{36}$$

where c^* is an estimate produced by an ordinary recursive least squares algorithm.

In real-time non-stationary signal processing it is appropriate to optimize the Lagrange function (15) using a gradient like procedure

$$\begin{aligned} & c(k) = c(k-1) - \eta_c(k)\nabla_c L(c,\lambda,k), \\ & \lambda(k) = \lambda(k-1) + \eta_\lambda(k)\partial L(c,\lambda,k)/\partial\lambda, \end{aligned} \tag{37}$$

or



$$
\begin{aligned}
& c(k) = c(k-1) - \\
& -\eta_c(k)\left(2\hat{y}(k)w(k) - \lambda(k-1)E\right), \\
& \lambda(k) = \lambda(k-1) + \\
& +\eta_\lambda(k)\left(c^T(k)E - 1\right)
\end{aligned} \tag{38}
$$

Vector c estimation error is calculated as

$$
\begin{aligned}
& \tilde{\theta}(k) = c - c(k) = \tilde{\theta}(k-1) - \\
& -\eta_c(k)\left(2\hat{y}(k)w(k) - \lambda(k-1)E\right)
\end{aligned}. \tag{39}
$$

A value of the step size parameter $\eta_c(k)$ that provides a maximum convergence rate can be obtained as the solution of the following differential equation

$$
\frac{\partial\left(\left\|\tilde{\theta}(k-1)\right\|^2 - \left\|\tilde{\theta}(k)\right\|^2\right)}{\partial\eta_c} = 0, \tag{40}
$$

where $\left\|\tilde{\theta}(k)\right\|^2 = \tilde{\theta}^T(k)\tilde{\theta}(k)$. The solution gives

$$
\eta_c(k) = \frac{2w^2(k) + \lambda(k-1)\left(c^T(k-1)E - 1\right)}{\left\|2\hat{y}(k)w(k) - \lambda(k-1)E\right\|^2} \tag{41}
$$

and the tuning algorithm for the output neuron G can be finally written as

$$
\begin{aligned}
& c(k) = c(k-1) + \\
& + \frac{2w^2(k) + \lambda(k-1)\left(c^T(k-1)E - 1\right)}{\left\|2\hat{y}(k)w(k) - \lambda(k-1)E\right\|^2}, \\
& \lambda(k) = \lambda(k-1) + \\
& +\eta_\lambda(k)\left(c^T(k)E - 1\right)
\end{aligned} \tag{42}
$$

If the Lagrange multiplier tuning loop maintains $c^T(k)E = 1$ the first expression in (23) takes the form of a well known in the artificial neural network theory Widrow-Hoff learning algorithm [8]

$$
C(k) = C(k-1) + \frac{w(k)\hat{y}(k)}{\left\|\hat{y}(k)\right\|^2}, \tag{43}
$$

that, in turn, is one of modifications of the delta-rule tuning.

7 Simulation results

As an example consider the simulation results for proposed forecasting algorithm, the initial time series is chosen as the superposition of three harmonics with frequencies $\omega = \begin{bmatrix}2.51 & 1.14 & 0.50\end{bmatrix}$ and amplitudes $A = \begin{bmatrix}0.8 & 1.5 & 1\end{bmatrix}$. The measurements are distorted by the random noise with uniform distribution in interval ± 0.5. The simulation results (Fig.6) illustrated that DFT extracts a fictitious harmonics whereas the proposed method ensures the stable estimation of frequencies and amplitudes.

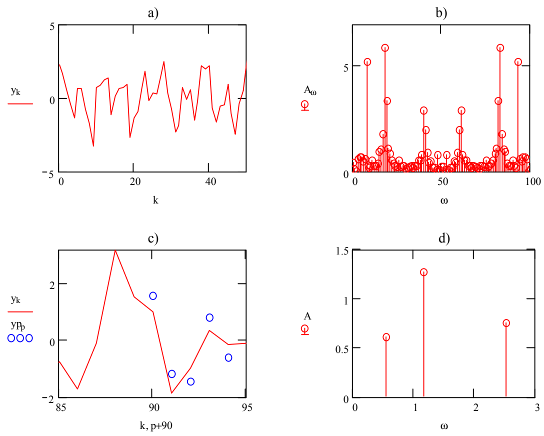


Fig. 6. Forecasting algorithm simulation
a) initial time series; b) DFT of time series;
c) time-series forecasting;
d) estimated harmonics frequencies.

The proposed method is applied for the problem of stock prices forecasting and buy/sell decision support [8]. The method is based on the idea of harmonic structure analysis of stock prices time series. Many samples of stock prices have a wave (non-periodic oscillating) structure so can be represent as a combination of the number of harmonics with unknown and changing frequencies and amplitudes.

The peculiarities of the method are the initial time-series decomposition on the slow (trend) and fast (oscillatory) components with the help of digital filters. The adaptive technique is used for model updating with the combination of detection of the days when the time series change its properties.

The proposed procedure of data processing and decision includes: identification of harmonic models using accumulated data (amplitudes and frequencies as well as necessary number of harmonics estimation) and short-term forecasting of stock prices and decision function construction in order to obtain the buy/sell decision recommendation in current day:

- *buy*, if the price is near the local minimum in current day and predicted price increases;
- *sell,* if the price is near the local maximum and the predicted price decreases;
- *hold* , in over cases.

Computer simulation has been fulfilled in order to evaluate the performance of the proposed method. The real data of stock prices (Fig. 7) has been used. The time series processed step-by-step (one step is one day), moreover in any current day only the previous data are assumed to be known so the actual behavior of stock trader is simulated. For each step the following calculations are performed:

- the initial time series is separated into the slow (trend) and fast (wave) components by the digital filtering algorithms (Fig. 8,10,12);
- the harmonic components of both trend and wave terms as well as first difference are estimated (Fig. 9,11,13) using the developed techniques. The four harmonics model is used;
- the short term (5 days ahead) forecast based on the estimated harmonic model (Fig. 14) is calculated (the solid line – real data, dotted line – forecast). In Fig. 15 the same curves are presented in enlarges scale;
- using the forecasted data, the decision rule is created. The decisions "buy" or "sell" is accepted if the estimated local minimum or maximum of predicted time-series is located near the current day respectively. The decision "hold" is accepted in any over cases. The fragment of decision sequence is presented at Fig. 16.
- passing on to the next steps the actual profit or losses are calculated. In Fig. 17 the accumulated profit from the first to current day is presented, i.e. the actual capital increment per one stock attained by the stock trading using the proposed forecasting method and decision rule.

The simulation results demonstrate the stable grows of the profit even in the case when the trend of stock prices has the tendency to the decreasing.

Conclusion

The proposed technique ensures the forecasting of non-periodic time series with wave structure. It can be treated as the development of structural approach for spectral analysis [7,8]. In such a way the critical point is the determining the optimal number of harmonics in the predictive model of wave component. Such a choice may be done using a multi-model approach, at that the adaptive algorithm of high level may adjust the model structure [17].

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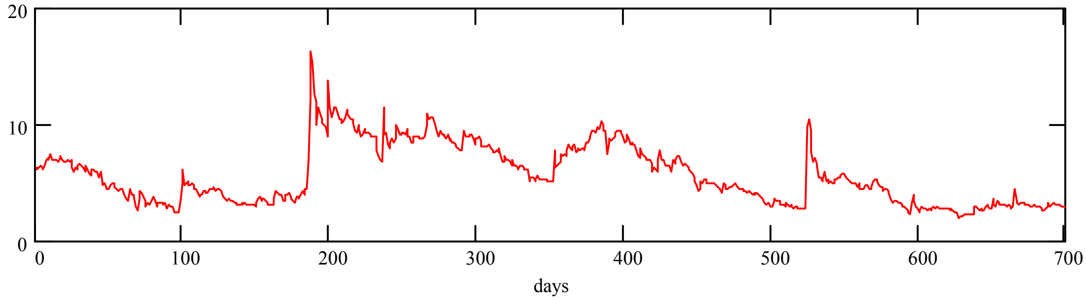

Fig 2. Stock prices

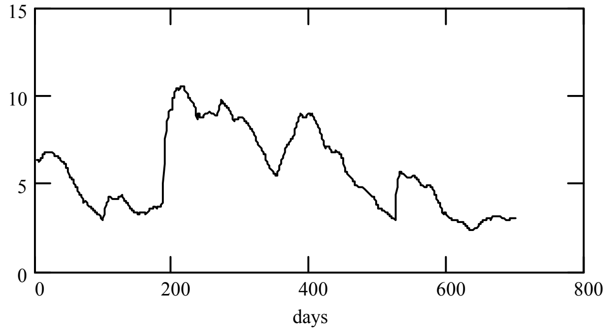

Fig 3. Trend (slow) component

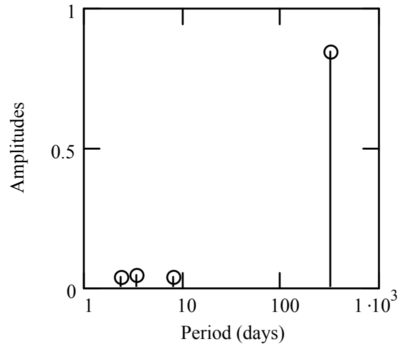

Fig 4. Trend's harmonics

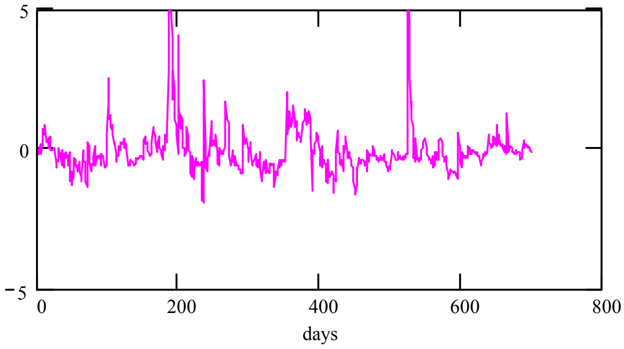

Fig 5. Wave (fast) component

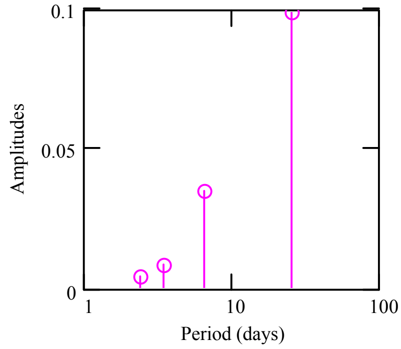

Fig 6. Wave's harmonics

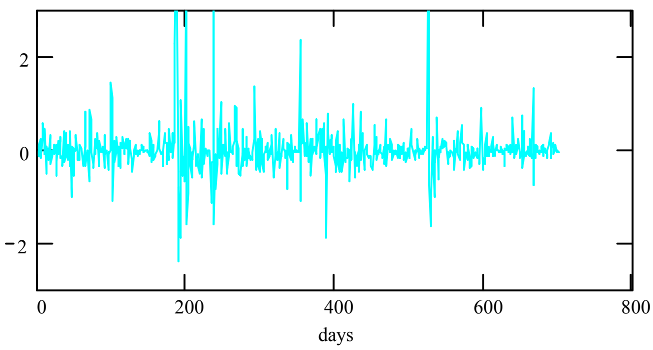

Fig 7. First difference

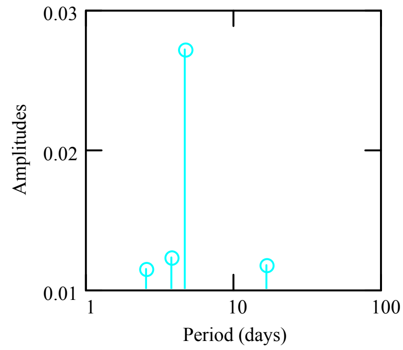

Fig 8. Difference's harmonics

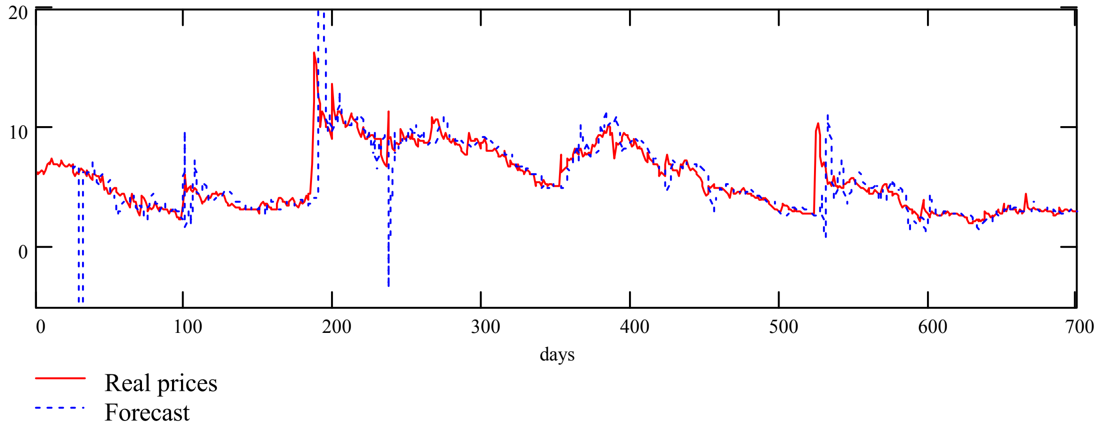

Fig 9. Five step ahead forecast

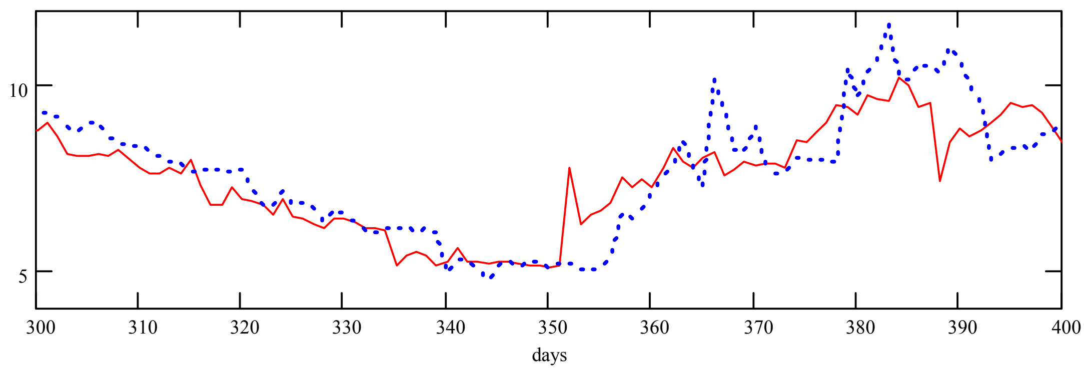

Fig 10. Forecast (zoom)

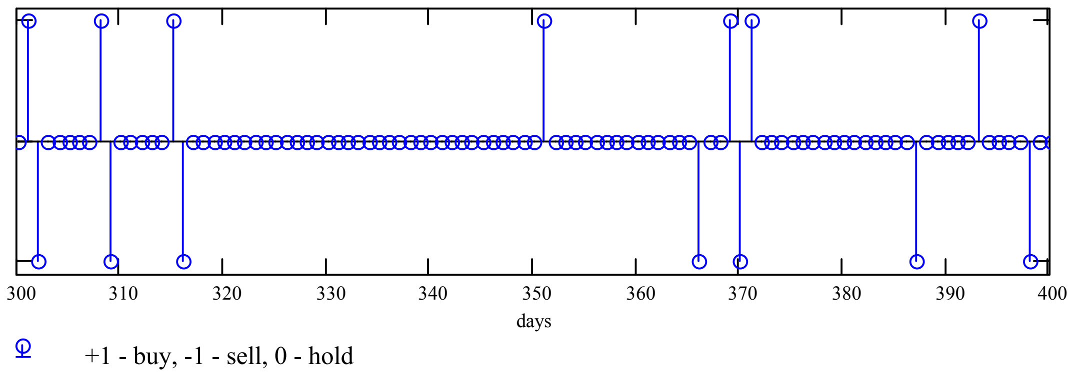

Fig 11. Buy/sell decision

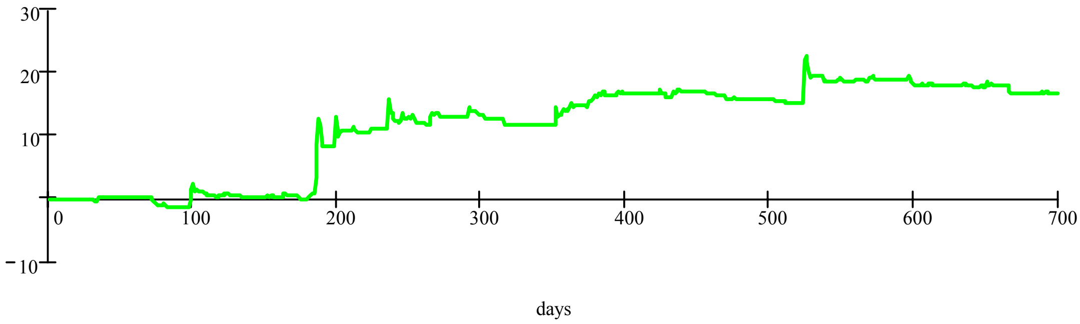

Fig 12. Current profit